Technical Report

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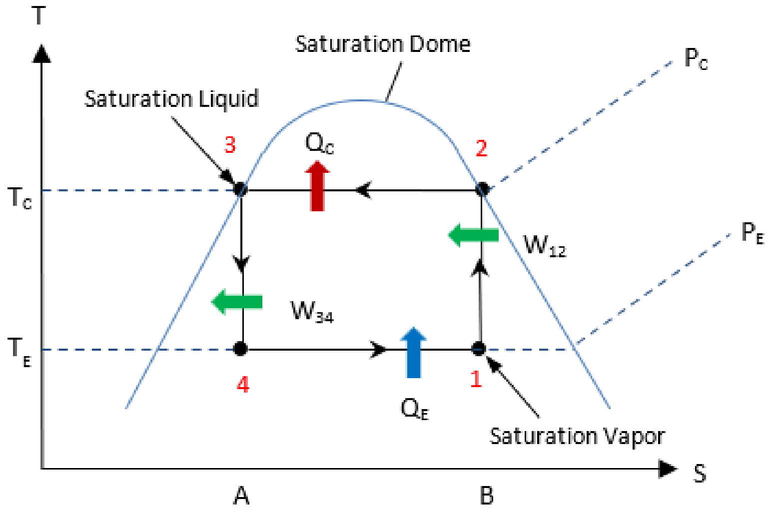
Saving the Planet One Building at a Time

Introduction to Heat Pumps



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1. OVERVIEW

1.1. Scope

The purpose of this paper is to provide the minimum necessary thermodynamics required to understand the concept of heat pumps; to explain *Energy Efficiency (η)* and discuss the relationship between energy efficiency and the *Coefficient of Performance (COP)*. We will show how it is possible for the efficiency of a heat pump to exceed 100%.

Heat pumps and refrigerators are systems that utilize thermodynamic cycles to "pump" heat from a cold source to a warm source. Heat pumps can be designed using a traditional *Vapor Compression Refrigeration System (VCRS)*. In this case, the primary means of work required to drive the cycle is provided by a mechanical compressor. Another type of heat pump called a *Gas Absorption Heap Pump (GAHP)* is similar to the VCRS but replaces the mechanical compressor by a thermal compressor.

The VCRS and GAHP also differ in the usage of *Renewable Energy (RE)* and *Non-Renewable Energy (NRE)* sources. Examples of renewable energy sources are ambient air, ground water or waste heat; which are "*free*". Examples of non-renewable energy sources are electricity, natural gas, coal; which are "*not free*" i.e., we need to pay for them.

It will be shown that the primary source of non-renewable energy consumption in the VCRS is due to the mechanical compressor in the form of electrical power. The principal advantage of the Gas Absorption Heat Pump GAHP with regard to a VCRC is that it replaces a mechanical compressor with a thermal compressor that utilizes both renewable and non-renewable energy simultaneously. In particular, it utilizes low grade ambient air and natural gas to drive the thermal compressor. The use of non-renewable energy as a supplementary energy source provides a means of reducing non-renewable energy consumption.

2. BRIEF REVIEW OF THERMODYNAMICS

2.1. Systems

The four major types of thermodynamic systems are shown in Figure 2-1. These are categorized by the permeability to three quantities; mass (*Δm*), work (*W*) and heat (*Q*).

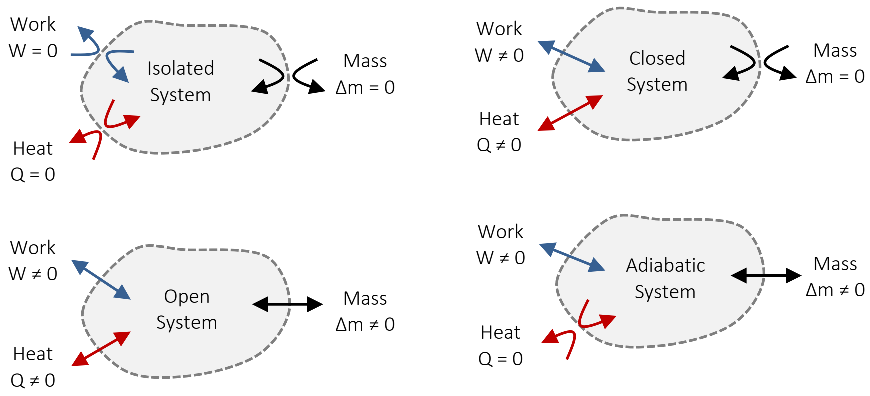
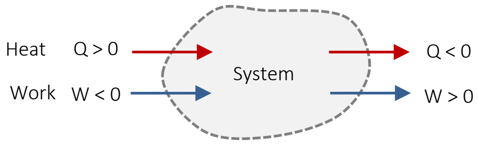


Figure 2-1 Permeability of systems cf. [1, p. 42]

Sign conventions for work and heat are show in Figure 2-2. The sign conventions shown in Figure 2-2 are adopted from early work on heat engines, where heat is input to get work out. However, in some texts heat and work into a system is positive and heat and work out of a system is negative [2, p. 37].

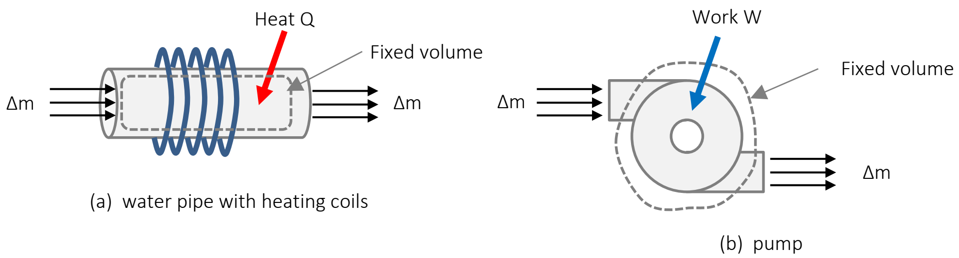


Heat added to system Q > 0
Heat removed from system Q < 0
Work done by surroundings on system on W < 0
Work done by system on surroundings W > 0

Figure 2-2 Signs for work and heat, based on heat engine convention

An *Isolated* system is not permeable to mass, work, or heat. It cannot interact or influence the environment. Once an isolated system reaches equilibrium, it will stay there forever without change. An idealized thermos at rest is an example of an isolated system cf. [3]. The universe itself is considered isolated since nothing exists beyond its boundaries.

An *Open* system is permeable to mass, work and heat and is also referred to as a *control volume* system. The boundaries of the control volume are not necessarily fixed and can change. However, the volume remains fixed. Common examples of control volumes are heat exchangers, pumps, turbines, and boilers. A section of a water pipe surrounded by a heater coil is an open system. Mass flows through the pipe which is a fixed volume, and heat may be added [3, p. 30].



(a) water pipe with heating coils

(b) pump

Figure 2-3 Open systems [3, p. 31]

A *Closed* system is permeable to work and heat but not mass and is also referred to as a *control mass* system. Although the mass of a closed system is fixed, its volume may be compressed or expanded. A fixed mass of gas in a cylinder with a piston is an example of a closed system. Work is input to the system to move the piston and compress the gas. The volume changes but the mass remains unchanged. Heat and/or work can be transferred to and from the system.

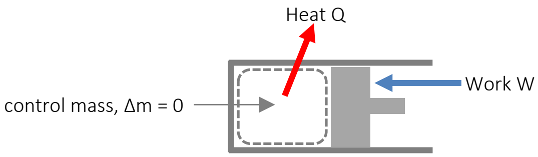


Figure 2-4 Piston: closed, control mass sysetm [3, p. 31]

An *adiabatic* system is a system which is not permeable the heat flow. Any barrier that prevents the flow of heat is known as an *adiabatic barrier*. A barrier that is permeable to heat is known as a *diathermal barrier*. A typical example of an adiabatic system is a standard thermos bottle. The walls are adiabatic barriers.

2.2. Properties

In thermodynamics we are generally interested in *properties* such as *mass*, *volume*, *temperature*, *pressure*, and *internal energy*. Properties depend only on the initial and final states of the system and so are said to be *memoryless* or *path independent*. A thermodynamic *state* is the set of properties of a system at *equilibrium* and at a particular instant of time. Properties of state do not depend on path the system takes to establish the equilibrium properties of the state. A property is not at equilibrium at each point during a process taking it form one state to another. The quantities are made conspicuous only when the system is in equilibrium.

Parameters can be either intensive or extensive. An *Intensive property* is a property whose magnitude does not depend on the amount of matter considered. These are the potential properties that can form energy gradients e.g., *pressure*, *temperature*, *electrical potential*, *density*, *chemical potential*.

These properties when existing in unequal equal amounts can cause an *energy gradient*, which in turn is the driving force for the flow of energy. Thus, these properties are sometimes termed *effort* or *intensities* variables. An *extensive property* is a property whose magnitude is dependent upon the amount of matter considered e.g., *energy*, *volume*, *charge*, *mass*. If you cut a box in half, its volume and mass get cut in half. These are the properties that will flow if exposed to an energy gradient and so these flow properties are sometimes called *flow* or *flux* or *extensities*.

If we take any extensive property and divide it by the mass of the system, we obtain a new intensive property denoted by the prefix *specific*. In the sense that this is independent of the mass in the system it may be regarded as an intensive property for the purpose of defining the state. For example, the volume (*V*) of a system is an extensive property. Dividing it by mass (*m*) gives us the volume per unit mass, known as the *specific volume* [3]

$$v = \frac{V}{m}\left[\frac{m^3}{kg}\right] \qquad 2.1$$

The specific volume is the reciprocal of the *density*, meters cubed per kg

$$\rho = \frac{1}{v} = \frac{m}{V}\left[\frac{kg}{m^3}\right] \qquad 2.2$$

Similarly, we can define the *specific energy* of a system, Joules per kg

$$e = \frac{E}{m}\left[\frac{J}{kg}\right] \qquad 2.3$$

Two states are identical if and only if all these properties are identical. Conversely, all properties characterizing the state of the system are *state functions*. A fundamental feature is that a variation of a state function when the system goes from one state to another depends *only on the initial and final states*, and not on the actual pathway followed during the transformation i.e., they are path independent. Mathematically, this means that the differentials of state functions are *exact differentials*. [4]

Common state variables (properties) are pressure, position, and volume, mass, and internal energy.

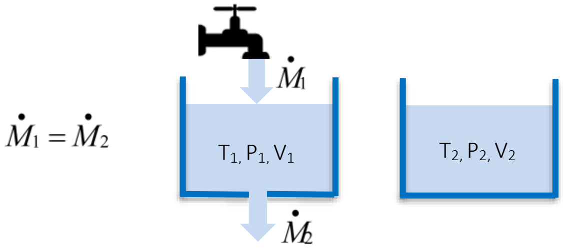
- *Internal energy (U)* ...... is the energy of the constituent microscopic particles. The potential energy of molecular bonds and kinetic energy of rotational, vibrational, or translational molecular motion. It is a property since its value depends only on the state of the system.
- *Temperature (T)* .... is a system property proportional to the systems molecular kinetic energy. In effect, temperature measures molecular kinetic energy to within a constant of proportionality.
- *Pressure (P)* .... is the normal force per unit area exerted by a substance on its boundary.
- *Mass (m)* .... is a measure of the amount of matter in an object. Mass is usually measured in grams (g) or kilograms (kg). Mass measures the quantity of matter and is invariant to location. Weight on the other hand is a force and is not invariant to location. For example, an object's mass is the same on the earth and the moon. However, an object's weight on the moon is one-sixth of your weight on the earth.
- *Volume* .... is a measure of how much space an object or substance takes up. Usually, measured in cubic meters (m^3).

2.3. Equilibrium and Steady State

The system under investigation can be described at any *instant* of time by a specific set of properties or state. A thermodynamic state need not be specified by all state properties, just those relevant to the analysis of interest. Several definitions are given below cf. [5].

- *Equilibrium* .... exists when in the absence of external forces, system properties do not change with time.
- *Steady State* .... a system that is interacting with its surroundings, but whose properties are not changing with time in steady state
- *State* .... is a set of properties of a system at *equilibrium* at particular instant of time
- *Phase* .... the term phase is used to denote the molecular constitution of a substance e.g., ice, water, and steam, respectively the solid, liquid and vapor phases of H_2O.

It is important to understand the difference between *steady state* and *equilibrium*. A system that is interacting with its surroundings, but whose properties are not changing with time is in *a steady state*. An isolated system with *constant properties* is at equilibrium. To illustrate, consider the following systems shown in Figure 2-5. In system (a) when we open the water tap the water level rises until the rate at which water flows out of the drain equals the rate at which it flows in, at which time the system has reached a steady state. That is, the mass of water in the sink remains constant even though there is constant flow in and out of it. In system (b) when we close the drain, turn off the tap and leave the sink isolated from the surroundings, it will be at equilibrium.



(a) Steady State (b) Equilibrium

Figure 2-5 (a) a system at steady state (b) an isolated system at equilibrium [3]

2.4. Duhem's Postulate

An important feature of a system is given by *Duhem's* postulate (1861-1916), which states that regardless of the number of phases, components or chemical reactions, the equilibrium state of a closed system for which one knows the initial mass of all components, is determined by *only two independent variables*. [4]

2.5. Process

A *process* is a series of actions that change the state of a system e.g., from state *a* to *b*. A complete list of properties describes the state of the system. If any property is changed the system shifts to a different state. Thus, we can depict any thermodynamic process on a two-dimensional graph by selecting only two properties showing how they vary during the process path. Any process, or series of processes, that results in the system being restored to its initial state is known as a *cycle*. For example, pressure (*P*) and volume (*V*) are frequently used as axes to describe the compression or expansion of a gas [6].

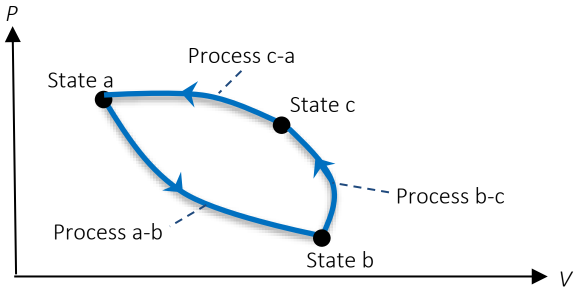


Figure 2-6 Processes *a-b-c-d* form a cycle

To define any process taking the system from state *a* to state *b*, we must calculate the quantity for each point along the actual path the process takes. This takes the form of an integral of the quantities from *a* to *b*. Unfortunately, in many cases the functional form of the process may not be known unequivocally during the process.

2.6. Heat and Work are Processes

Contrary to popular belief, work and Heat are not properties but processes. *Work (W)* is energy transferred across a system boundary by means other than Heat. So, it's sometimes called *adiabatic work*. *Heat (Q)* is energy transferred across a system boundary by virtue of a temperature gradient.

In essence thermodynamics is the study how (1) heat and work transfers influence the properties of a system or conversely (2) how the changes in properties influence the heat and work transfers. Recall that parameters of a fluid such as pressure, temperature, volume, density, mass are properties. But *Q* and *W* by their very nature can never be considered as properties.

- Q and W never exist in an object or fluid - they occur at the boundary of a system and are transient in nature.

Mountain Analogy for work (*W*) [5]

A mountain with two different routes is seen in Figure 2 7. The simple route is path 1, while the difficult path is path 2. If you choose the straight way, you run the risk of having no track to follow, having to climb steep mountain slopes with boulders and more. To reach the summit, the hiker will need to put in a great deal of energy. Path 1 is a well-traveled, well-maintained trail with a gentle slope. To get to the peak, far less energy is needed. In either case, when the hikers reach the summit, the height they have climbed is the same 1,000 feet.

The starting and ending elevations of H1 = 0 ft and H2 = 1000 ft are system properties of the mountain; the path chosen does change their values. However, the amount of work depends on the path chosen. That is, the system has memory. At every instant the hiker moves he is at a new elevation, but the system is not in equilibrium until he reaches the top and rests. At that point there is no more work being expended and the system is in mechanical equilibrium.

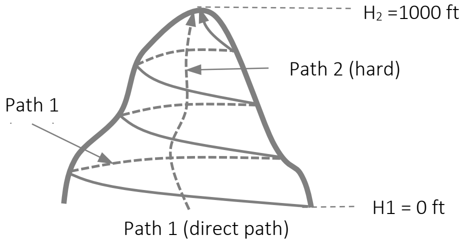


Figure 2-7 Mountain analogy for work

Example: Blocks in space – position is a property

The 2-dimensional coordinate of an object shown in Figure 2-8 is a property. The block is initially at position (x_1, y_1). The block is moved to position (x_2, y_2). If you are asked to measure the position of the block at position 2. You simply measure the *x* and *y* coordinate of the block. It does not require a knowledge of the starting position or path. [3, p. 31].

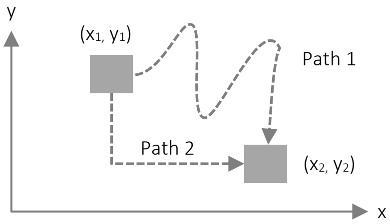


Figure 2-8 Work is not a property cf. [3, p. 31]

Example: Rain Analogy for heat (Q)

Figure 2-7 shows water vapor in a cloud and liquid water in a lake below. The amount of water in the cloud and lake m_1 and m_2 are measurable properties that do not require any memory. When water vapor condenses, it falls and although it's still water, it's now also rain. The rain is not stored in the cloud or lake only water is stored. Once the rain hits the surface of the lake it is no longer rain but just water. The water transfer is not a property but a process involving the transfer of mass by way of a gravitational potential energy gradient.

At every instant the rain falls, the cloud and lake have a new amount of water, however the system is not in equilibrium until the rain stops. At that point there is no more rain (water in transit) and the system is in equilibrium expressed by the new values for the amount of water in the cloud and lake.

Now compare it with heat transferred between a hot and cold object by virtue of a temperature gradient. The hot body has higher internal energy relative to the cold body. The internal energies correspond to the water stored respectively in the cloud and the lake and the heat transfer corresponds to the rain [5].

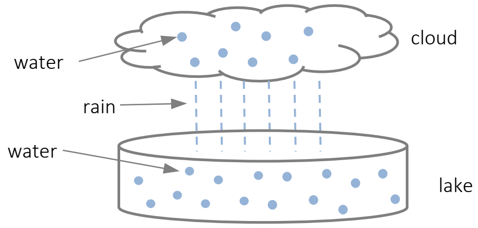


Figure 2-9 Rain analogy for heat

Example: Temperature is a property

A block is initially at temperature T_1 and is heated to T_2. Temperature is a property since temperature has no memory; to determine the final temperature does not require a knowledge of the initial temperature. However, heat is not a property. To find the amount of heat added you need to know the initial temperature as well as the final temperature. However, knowing the temperatures alone will not tell you how much heat was added. Temperature can rise due to heat being added to the fluid or it can be raised due to mechanical work being done on the fluid, such as rotating a paddle wheel, this was the classic experiment that Joule used to determine the mechanical equivalence of heat.

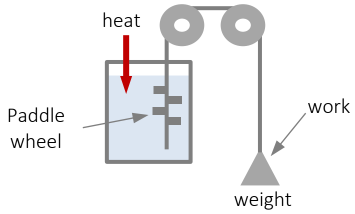


Figure 2-10 Heat is not a property, Joules paddle wheel cf. [3, p. 31]

2.7. Fundamental Processes

There are several common processes encountered in thermodynamics

- *Adiabatic* .... Not permeable to heat flow i.e., no heat transfer in or out, $Q = 0$. Change in internal energy U is equal to the work W applied to the system i.e., $\Delta U = -W$. For expansion, $W > 0$ and $\Delta U < 0$. For compression, $W < 0$ and $\Delta U > 0$
- *Isochoric* .... A constant volume process i.e., no PV work, $W = 0$. Change in internal U energy is equal to the work applied to the system i.e., $\Delta U = Q$
- *Isobaric* .... A constant pressure process i.e., $W = p (v_2 - v_1)$
- *Isothermal* .... A constant temperature process. It is not generally true that $\Delta U = 0$ in an isothermal process. An ideal gas by definition has no interactions between particles, no intermolecular forces, so pressure changes at constant temperature does not change internal energy. Real gases have intermolecular interactions, attractions between molecules at low pressure and repulsion at high pressure. Their internal energy changes with change in pressure, even if temperature is constant. For an ideal gas, in an isothermal process, $\Delta U = 0 = Q - W \therefore Q = W$

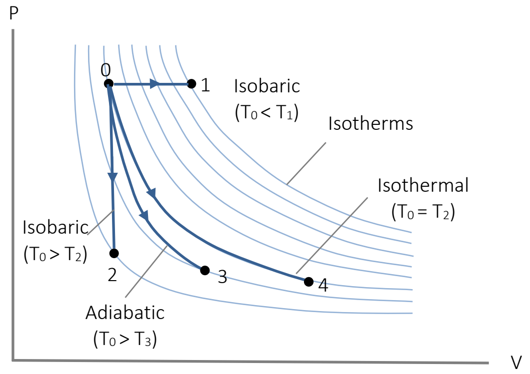


Figure 2-11 process from state 1 to state 2 in *T-v* space: actaul and hypothetical *path 1*.

2.8. Second Law of Thermodynamics

All *spontaneous activity* in the material world, are examples of the second law because they involve energy dispersing spontaneously. That is, they occur without adding energy from outside the system.

A macroscopic view of the second law may be understood in terms of *energy gradients* and that the second law makes conspicuous natures tendency to spread or disperse energy. This viewpoint is intuitive since it correlates with physically observable processes we see every day. For example, we observe that energy flows from a higher to lower temperature (heat flow), from a higher pressure to lower pressure (expansion), from a higher voltage potential to a lower voltage potential (electric current) and from higher gravitational potential to lower gravitational potential (falling objects). Even chemical reactions proceed spontaneously from higher concentrations of molecular bond energy to lower bond energies. In some cases an initial activation energy is required to start a chemical process to overcome initial constraining energies.

These observations lead one to conclude that the second law of thermodynamics is simply a statement that if left unconstrained, energy will flow from high to low energy concentration but never the other way. Or equivalently, nature has an unending propensity to spread and distribute energy equitably into the largest extent possible within the constraints of the system.

- The second law tells us which processes in nature can occur spontaneously and which cannot.

Natures propensity to spread and distribute energy is an observed fact. However to understand "why" nature behaves this way, can only properly be understood by in vestigating the microscopic nature of matter. In this view we would discover that natures driving force is based simply on statistics and the probability that energy will occupy a macro state with the largest number of available microstates. Similar to rolling dice, the probability that you roll 1000 dice and they all come up heads is unlikely. It turns out the most likely state is when there are an equal number of heads and tails. Thus the dice are most likely to be in the state with the most possible available outcomes. Comparing this to a chamber filled with a gas, it is highly unlikely that the gas molecules would spontaneously all reside in the left hand side of the chamber. However, they are most likely to appear evenely distributed throughout the box maximizing the number of available miscrostates. Thus, statistics is mother natures way of driving sponaneous processes and is the underlying cause for the second law of thermodynamics.

Implied in macroscopic observation is that a system with high energy concentration relative to a low concentration creates a *potential energy gradient*. Recall, in mechanics, potential energy is associated with the energy of position. We take liberties here to generalize the definition to include any difference between energy concentrations. In this way we can cast all energy gradients, positional or otherwise in the same mathematical framework. It should be noted that Helmholtz (circa 1860) used the term equipotential energy in this broader sense.

This potential energy gradient can liberate some of its energy as mechanical work or heat as it flows from high to low energy concentrations. The amount of energy available to be liberated as work is called *free energy*, since it is free to perform work. As the energy flows from the high energy potential to low energy potential, energy is transformed into the energy of motion which we shall call energy *flow* or *flux* or *current*. The flux of energy necessarily depletes the potential energy gradient until all the potential energy has been dissipated. At that point all energy is at the same potential and is referred to as *bound energy* or *equipotential energy*. It is bound since is no longer available to do work or produce activity.

Energy gradients may be partially depleted, so this energy is sometimes referred to as *degraded energy* because it is less useful. Remember *all activity requires the flow of energy* and *all energy flow requires an energy gradient* or potential energy source and the *effect of every energy change is less useful energy*. cf. (Watson, 2010)

We may express the second law by stating that nature tends to *minimize potential energy*. We are not saying nature seeks to minimizes energy since by the first law, in an isolated system energy is conserved. It cannot be maximized or minimized but remains constant. However, nature does seek to *minimize potential energy* that are manifestations of *energy gradients*. Thus, we can define the second law intuitively from a macroscopic point of view in a number of equivalent ways.

- *Second Law* .... The driving force of all activity is nature's unending propensity to spontaneously flow from a high concentration of energy to a lower concentration of energy. Molecular motional energy if unconstrained will *spontaneously disperse*.
- *Second Law* .... Every activity (energy change), if unconstrained, will spontaneously convert some or all of the *free energy* (more concentrated) into *bound energy* (less concentrated) and the free energy is lost forever within an isolated system. i.e., without the addition of external energy from outside the system
- *Second Law* .... Nature if unconstrained seeks to minimize potential energy.

In an isolated system an energy gradient can expend its free energy to do work and/or create bound energy. However, the system cannot by itself increase its free energy. If two blocks one hot and one cold, are brought in contact, the thermal gradient allows the flow of free energy from the hot to the cold block, but not the other way around. Once the blocks have come to an equilibrium, both are at the same temperature and no thermal gradient exists. The blocks cannot spontaneously reverse the process and restore the initial state of hot and cold. It's a *one-way street*. Thus, the second law is as Arthur Eddington referred to is *time's arrow*. It indicates the direction of time and activity in the world. For example, water always spontaneously flows downhill but never uphill, unless aided by external means such as a pump which requires an external electric energy gradient in the form of a charge potentential i.e., voltage.

2.9. Entropy

Another extremely important but often confusing concept is *entropy*. In order to effectively analize the thermodynamic cycles involved in heat pumps and refrigerators, it is instructive to review the concept of entropy. First lets state for the record that unlike the popular misconception[1], "*entropy does NOT say everything tends toward disorder. It is NOT a universal law of messiness. It is only about energy changes*" (Watson, 2010).

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[1] This popular misconception that entropy is disorder, is even to be found in textbooks on Thermodynamics. However, in recent years, many authors have recognized this pedagogical error and have been shifting towards an energy gradient paradigm [8].

The definition of Entropy from a macroscopic point of view in terms of energy dispersal and gradients can be expressed in a number of equivalent ways cf. [7], [8]

- *Entropy* .... is a measure of the amount of free energy converted to bound energy during a thermodynamic process at a given temperature
- *Entropy* .... measures on a macroscale a spontaneous dispersal of energy from a higher concentration to a lower concentration of energy at a given temperature

We can apply these definitions to a few simple examples

Example 1: Heat flow from hot to cold

"Consider two boxes A and B where A is at temperature T_A and B is at temperature T_B. Assume that $T_A > T_B$. Initially, on average, the molecules in A are moving with higher velocity (higher kinetic energy) then the molecules in B. Now bring the boxes together separated by a thin membrane. The molecules in A start to strike the membrane losing some momentum (velocity) to the molecules in B just adjacent to the membrane. These in turn strike molecules adjacent to them and so on. The process continues until all the molecules, on average, are moving with the same momentum (velocity) in both containers" [8, p. 15].

We can guess what has happened based on our definition of the second law. The energy in A contained a higher concentration of the kinetic energy than B. An energy gradient existed. When brought into contact with B, box A started dispersing some of its energy to B and in the process reducing the energy gradient. It continues spreading its energy until the total energy in the combined system A and B is spread equitably to the largest extent possible, reaching equilibrium. At equilibrium the average kinetic energy i.e., temperature, is the same for A and B and the energy gradient or driving force has been reduced to zero. This is the second law in action. No disorder to worry about.

Other confusions about equilibrium and maximizing entropy result from common statements about the second law of thermodynamics states indicating that

- *Equilibrium* .... If left unconstrained, a system will spontaneously change until thermal equilibrium is achieved and entropy is maximized.

Again, this is unintuitive. What does it mean for entropy to be a maximized? However, if we think of entropy as the measure of *free energy* existing by virtue of an energy gradient, then intuitively we can see that the system spontaneously disperses energy, and in the process, diminishes the gradient proportionately. The process continues until a state of equilibrium is reached, where the energy gradient is zero and the free energy has been completely converted to bound energy.

Thus, a better definition of equilibrium is:

- *Equilibrium* .... A system, if unconstrained, spontaneously tends toward equilibrium where all energy gradients are zero or have stopped changing due to constraints

Example: Water Flow

Given a barrel filled with water to a height h_1. You open a spigot and the water flows into a bucket with water level initially at $h_2 = 0$. An energy gradient exists due to the height differential between the water in the barrel and the bucket. Once the spigot is opened, water flows due to the potential energy gradient of the water $h_1 > h_2$. As the water flows, the energy gradient is diminished as the water level in the bucket rises and the level in the barrel decreases. During the flow of water when $h_1 > h_2$, work can be done by forcing the water through a turbine, so the energy is free to do work. When the water level in the barrel is at the same level as the bucket $h_1 = h_2$, the water stops flowing. All the free energy has been converted to bound energy and is no longer available to do work i.e., turbine stops.

Heat-Death of the Universe

Now if we consider that the universe is an isolated system, this implies that someday all the free energy in the universe will be completely converted to bound energy. This is the so-called *heat-death of the universe*. Sounds pretty ominous. However, as [7, p. 111] points out "*We should all give thanks that nature spends free energy, because without the steady increase of entropy, there would be no activity in the universe. Everything would be at absolute zero degrees. No atomic or molecular motion …"*.

- Without an increase in entropy there would be no life!"

2.10. Postulates of Kelvin and Clausius

Historically, the second law and entropy were discovered by studying the limitations of heat engines. That is, systems like the steam engine, are designed to transform heat into work, even though energy is conserved. These systems use heat, typically combusted fuel and convert it to mechanical work and expel exhausted heat back to the environment. The system undergoes a thermodynamic cycle which starts at a given thermodynamic state and returns to the same state. The processes of heat engines were assumed to obey the following postulates: (Sekerka, 2015, p. 31)

Postulate of Kelvin:

- A transformation whose only final result is to transfer into work, heat extracted from a source which is at the same temperature throughout is impossible.

Postulate of Clausius:

- A transformation whose only final result is to transfer heat from a body at a given temperature to a body at a higher temperature is impossible."

These postulates, although true, provide no physical intuition, they are really *a consequence* of the second law and should not be used as a fundamental definition. The defining equation for entropy used to formulate the Clausius or Kelvin-Plank form of the second law is based on Clausius definition of entropy.

$$dS = \frac{\delta Q_{rev}}{T} \qquad \text{Clausius definition of a change in entropy} \qquad 2\text{-}4$$

Where S is the entropy change of a system at absolute temperature T (Kelvin) during a reversible heat process and is the energy added to the system. As (Leff, 1996) points out "*It is gross understatement to say that students have difficulty grasping the traditional approach*." First consider that it requires a knowledge of reversable processes and heat engines, this is putting the cart before the horse. Reversable processes and heat engines should be studied after one is introduced to entropy. Not the other way around. Equation 2.5 provides no intuitive physical meaning. And even more confusing is the notion that entropy can change for irreversible processes even when $Q = 0$. What do you do then? Now, let's return to our working definition of entropy, armed with a solid inuitive understanding of entropy as it relates to free and bound energies. The *Clausius Inequality* states

$$dS = \oint \frac{\delta Q_{rev}}{T} = 0 \qquad \text{reversible processes} \qquad 2.5$$

$$dS = \oint \frac{\delta Q_{rev}}{T} < 0 \qquad \text{Irreversible processes} \qquad 2.6$$

The integration is performed over a closed path and is always less than or equal to zero. The equality holds for internally reversible cycles and the inequality holds for cycles with ireversabilities i.e., all physically realizable systems contain dissipative forces and are irreversable.

$$dS = \frac{\delta Q_{rev}}{T} \qquad \text{Where } S \text{ is entropy (reversable)} \tag{2.7}$$

Example 2: Free expansion gas

One example [8, p. 15] that causes confusion is when a gas expands spontaneously and isothermally into an evacuated chamber. Since $Q = 0$ we can't use 2.7, so one would be tempted to say that there was no change in entropy. This is a common mistake assummed by many students. However, based on the energy spreading definition of the second law and entropy, we recognize that the process was spontaneous therefore, entropy increases. We can also see that the gas went from concentrated to less concentrated, so again, inuition says that entropy has increased. In traditional thermodynamics course you find out that although $Q = 0$, you need to use the *ideal gas law* to determine the resulting change in entropy which is $\Delta S = -R\,T \ln V_2/V_1$, where R is the universal gas constant, V_1 is the initial volume and V_2 is the final volume.

$$\Delta S = -RT \ln\left(\frac{V_2}{V_1}\right) \qquad \text{Change in entropy for free expansion of an ideal gas} \tag{2-8}$$

This example further illustrates the shortcomings of the traditional definition of entropy as chaos or by relying exclusively on equation 2.7, for our understanding of entropy. We need to be judicious in our application of equation 2.7 and use it only when it makes sense to do so.

We can use the Claussius definition of entropy change to show that heat can spontaneously only flow from hot to cold and not the other way around.

Example – Show that heat can spontaneously flow from hot to cold but not from cold to hot

Give two blocks, one colder at T and another hotter at $T + \Delta T$. The heat out of one block equals the heat into the other block, thus Q is the same for both blocks. The change in entropy from the colder block is greater than the change in entropy for the hotter block due to T and $T + \Delta T$ in the denominators respectively.

$$\Delta S_c = \frac{Q}{T} \qquad \Delta S_h = \frac{Q}{T + \Delta T} \quad \overset{implies}{\rightarrow} \quad \Delta S_c > \Delta S_h \tag{2-9}$$

Therefore, the spontaneous change in entropy can occur from hot to cold but not from cold to hot.

$$\Delta S_{c \to h} = -\Delta S_c + \Delta S_h < 0 \qquad \text{Cold to Hot } \textbf{not allowed} \text{ `(spontaneous increase in free energy)} \tag{2-10}$$

$$\Delta S_{h \to c} = \Delta S_c - \Delta S_h > 0 \qquad \text{Hot to Cold } \textbf{allowed} \text{ (spontaneous decreases in free energy)} \tag{2-11}$$

We see that a spontaneous flow from hot to cold results in an increase in total system entropy (decrease in free energy) and is allowed based on the second law. However, a spontaneous flow from cold to hot results in a decrease in system entropy (increase in free energy) and is not allowed based on the second law.

3. HEAT PUMPS

3.1. What is a Heat Pump?

With a brief review of thermodynamics under our belt, we are armed with the tools needed to analyze the HVAC refrigeration cycle and heat pumps.

The goal of a heat pump is to make heat flow from a cold source at temperature T_C to a hot source at temperature T_H. We should note that a refrigerator is a heat pump, however, convention has adopted the notion that if the space to be conditioned is the cold space, it is called a refrigerator; if the space to be conditioned is the hot space, it is called a heat pump. But in either case, heat is being pumped from cold to hot, so they are technically both heat pumps. We shall stick with convention here and use refrigeration to imply the conditioned space is the cold space.

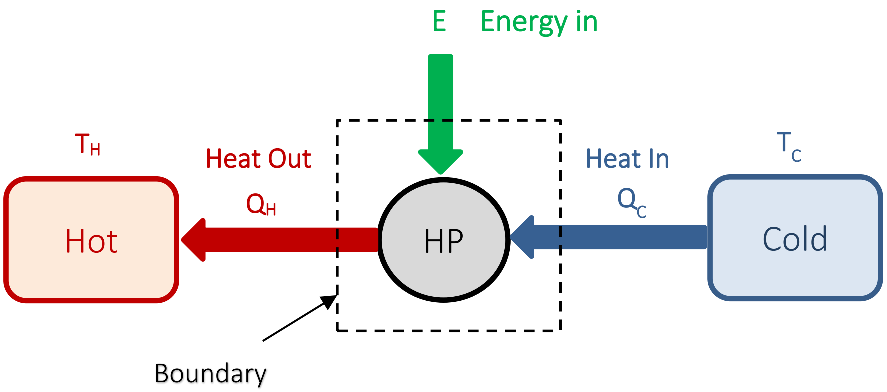


Figure 3-1 Simplified diagram of a Heat Pump

The first point to make is a result of the *entropy* of the system. Entropy is a property of nature resulting from observation over hundreds of years. It cannot be proved from first principles or more fundamental laws and is similar to an axiom in mathematics. As we describe in the previous section, entropy is a measure of the *spontaneous* molecular *energy dissipation* of a system. Adding energy to a closed system causes molecular particles of the system e.g., atoms, electrons, molecules by way of vibrations to increase their kinetic energy which results in an increase in temperature. Higher energy also means a higher probability that an atomic particle occupies a higher energy level. Thus, the total number of energy states that can be occupied increases; resulting in a net increase in the degree energy is spread out to the widest extent possible. Thus, higher temperatures are associated with higher potential entropy content.

A consequence of the second law of thermodynamics is the fact entropy can only increase but never decrease spontaneously i.e., *heat can flow spontaneously from hot to cold, but not from cold to hot*. The key word is *spontaneously*. This implies that heat can flow from a hot source to a cold sink without requiring external energy. On the other hand, in order to force heat to flow from cold to hot requires external energy.

$$T_H \overrightarrow{spontaneous}\ T_L \qquad \text{From second law of thermodynamics} \qquad 3.1$$

The second point is based on the first law of thermodynamics which is a statement of the conservation of energy. That is, energy cannot be created or destroyed; it can only be moved or transformed. Given the system is enclosed by a boundary shown in Figure 3-1, as a result of the first law of thermodynamics, we observe that the total energy leaving the system cannot exceed the total energy entering the system.

$$Q_H \leq Q_C + E \qquad \text{From the first law of thermodynamics} \qquad 3.2$$

3.2. Efficiency

The *absolute efficiency* (η) of the heat pump system in Figure 3-1 can never exceed 100%. As we have noted, this is a consequence of the first law of thermodynamics which states that energy cannot be created or destroyed.

$$\eta = \frac{Q_H}{E} \leq 100\% \qquad \text{Absolute Efficiency} \qquad 3.3$$

However, we are not interested in the absolute system efficiency. We are interested in the efficiency defined by the heat delivered to the hot space, divided by the amount of non-renewable energy used to move the energy from the cold to hot space. Thus, the efficiency of a heat pump and refrigerator are defined as:

$$\eta_R = \frac{Q_C}{E_{NR}} \qquad \text{Refrigerator efficiency} \qquad 3.4$$

$$\eta_{HP} = \frac{Q_H}{E_{NR}} \qquad \text{Heat Pump efficiency} \qquad 3.5$$

We can modify our heat pump diagram to show the separation of external energy into renewable E_R and non-renewable E_{NR}.

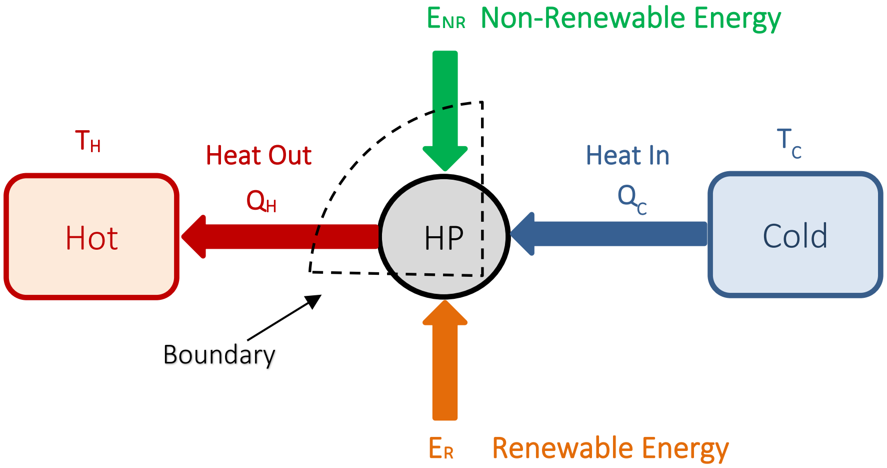


Figure 3-2 Heat Pump showing renewable and non-renewable energy inputs

As will be shown later, the efficiency for refrigerators and heat pumps always exceeds 100%. In the refrigeration industry, in order to avoid confusion between the total efficiency (3.3) and non-renewable efficiency (3.5), they adopted a term called the *Coefficient of Performance COP*, which is simply the *non-renewable efficiency* described above (3.5).

However, some folks do not recognize COP as a true efficiency, which it is. Another twist is that the Heat Pump industry has in many cases reverted to the term efficiency in their specifications. Even the state of CT in their Gas Heating Rebate document [9], states that a Heat Pumps must exceed 100% efficiency to be eligible for energy incentives. The lesson to be learned; is that when dealing with efficiencies it is important to identify what quantities are being referenced.

There is a point of confusion that often arises[1]:

- *How can a heat pump have an efficiency > 100%, while a Boiler has an efficiency of <100%? Doesn't this violate the first law of thermodynamics?*

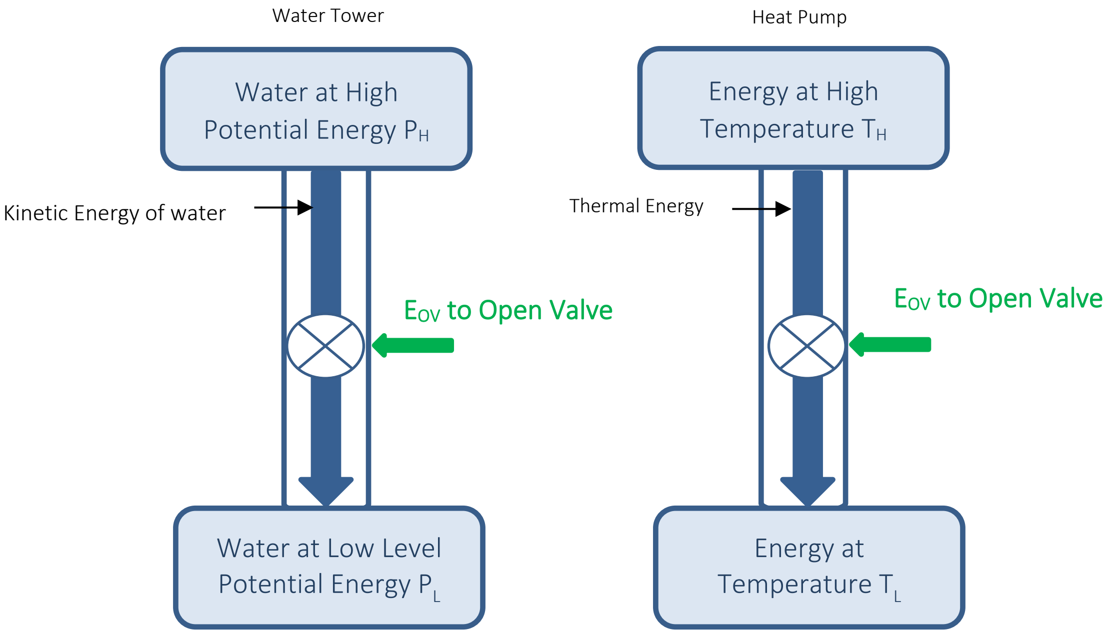
The answer is simple. It is true that the efficiencies of a boiler or gas-pack can never exceed 100% and are typically about 80% to 97%. It is also true, as given by equations 3.8 and 3.9, that the efficiencies of a refrigerator or heat pump exceed 100%. The reason the heat pump efficiency exceeds 100% can be understood by reference to the Hydro-Thermal Analogy in Figure 3-3. For the boiler or gas pack, the input work must be *transformed* to output energy. By the first law of thermodynamics this process must be less than 100%.

On the other hand, in the case of a heat pump; the input energy is only being used to turn the valve, it is not transformed into output energy. Opening the valve simply allows nature, by way of temperature differentials, to *move energy* from high temperature to low temperature spaces. The key words are *transformed* and *move*.

Heat pumps facilitate movement of energy while Boilers transform energy

3.3. Hydro-Thermal Analogy

Based on the discussion above, *a heat pump does not violate the laws of thermodynamics*, even with non-renewable efficiencies greater than 100%. The reason is that the heat transferred between the cool and warm spaces is not part of the non-renewable energy used in the compressor. In other words, the work input is never transformed into heat but helps guide the transport of energy from one place to another. To understand this, we introduce the hydro-thermal analogy shown below.



(a) Kinetic Energy of water (b) Thermal Energy of heat

Figure 3-3 Hydro-Thermal Analogy

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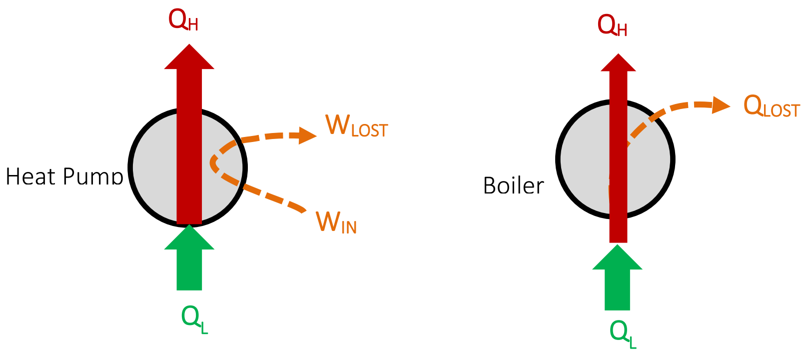
[1] I was asked this question by the utilities when applying for incentives for Gas Absorption Heat Pumps. It was for that reason I wrote this white paper.

In Figure 3-3a, it is clear that not much external energy is required to turn the valve on, allowing the natural force of gravity to generate large water flow. Depending on the mass flowrate of the water, the ratio of water flow (kinetic energy) relative to the input mechanical energy used to turn the valve can be enormous. Likewise, in the heat analogy Figure 3-3b, it does not require much energy to turn the valve and allow energy to flow from a high to low temperature source. Thus, we see temperature may be thought of as potential thermal energy. In fact, heat is defined as the process by which energy is transferred by temperature difference. It should be no surprise that moving heat/water from low temperature/potential to high temperature/potential would require external energy.

3.4. Heat Pump vs. Boilers

The energy flows into and out of a thermodynamic heat pump and a boiler are compared in Figure 3-4. These diagrams illustrate the fact that Heat pumps facilitate movement of energy while Boilers transform energy.



(a) Heat Pump: $Q_H \le Q_L + W_{in} - W_{lost}$

(b) Boiler: $Q_H \le Q_{lost}$

Figure 3-4 A Heat Pump vs. a Boiler

3.5. Heat Pump and Refrigerator COP

From the flows shown in Figure 3-4, we define the efficiency or COP of a heat pump as:

$$\eta_{HP} = COP_{HP} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - \frac{Q_H}{Q_L}} \qquad \text{Efficiency of a Heat Pump} \tag{3.6}$$

The desired output in a refrigerator is Q_L so the efficiency of a refrigerator is

$$\eta_R = COP_R = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L} = \frac{1}{1 - \frac{Q_L}{Q_H}} \qquad \text{Efficiency of a Refrigerator} \tag{3.7}$$

By the first law of thermodynamics, $Q_H > Q_L$ therefore the value of η_{HP} and η_R are always greater than 100%. This is in contrast to the absolute thermal efficiency (3.3) which can never exceed 100%. From equations 3.6 and 3.7 we can show:

$\eta_R > 100\%$	Efficiency of a Heat Pump and Refrigerator	3.8
$\eta_{HP} = \eta_R + 100\%$	Efficiency of a Heat Pump vs. Refrigerator	3.9
$COP_{HP} = COP_R + 1$	COP of a Heat Pump vs. Refrigerator	3.10

3.6. T-S Diagram

An extremely important diagram used extensively in thermodynamics and HVAC system design is the *temperature-entropy (T-S) diagram*. A *T-S diagram* for an arbitrary fluid is shown in Figure 3-5. A *T-S* diagram is characterized by a *vapor dome*. The area to the left of the dome is the *liquid region*, to the right is the *vapor region* and the area under the dome is the *liquid-vapor region*. The *critical point* is the end point of the saturation dome where there is no change of phase (liquid to vapor or vapor to liquid) for the fluid when pressure is increased or if heat is added. Inside the dome and to the left of the critical point is the *saturation curve*.

Points on the saturation curve such as “A” are saturated (100% liquid). Points on the vaporization curve such as “B” are vaporized (100% vapor). Points under the dome and between saturation and vaporization curves such as “C” are a mixture of liquid and vapor. Points closer to the left have a higher ratio of liquid to vapor while points closer to the right have a higher ratio of vapor to liquid. The *ratio of liquid to vapor* is called the *fluid quality*.

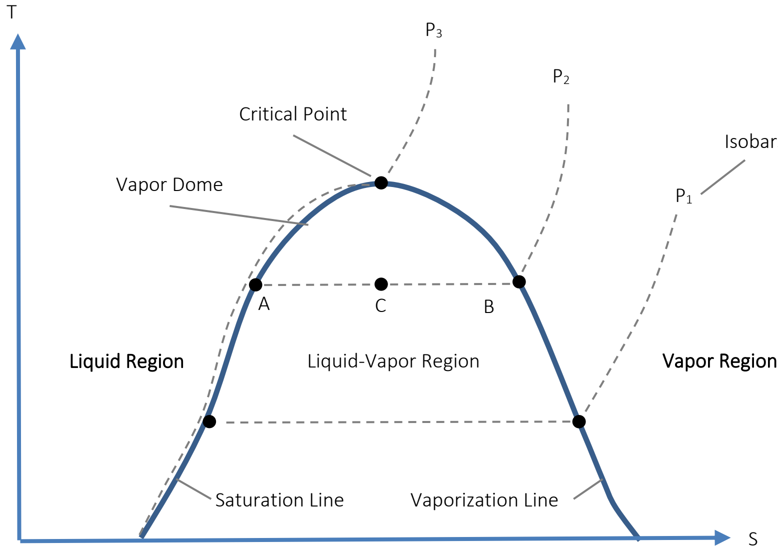


Figure 3-5 T-S diagram showing saturation dome and isobars

From

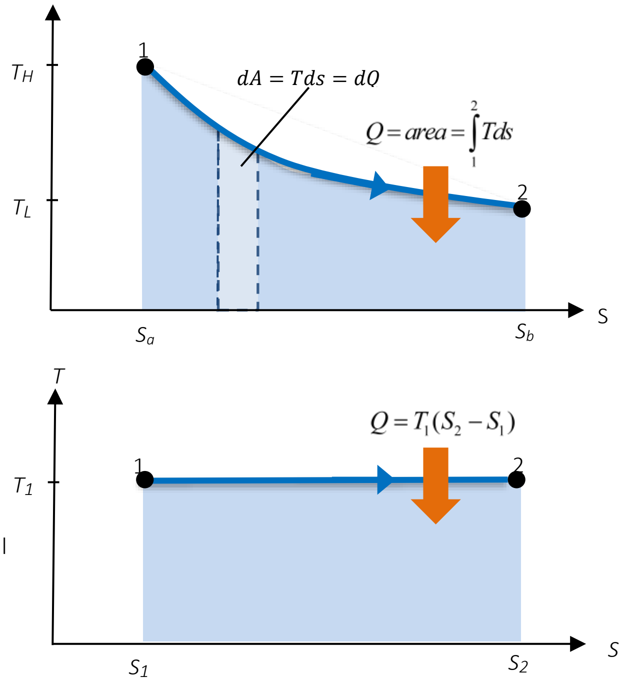
Figure 3-6 (a) we see that on a *T-S* diagram the area under the curve for the reversible process from point *1* to *2* is equal to the heat transfer during that process [10, p. 344]. The heat is into the system since entropy is increasing from S_1 to S_2 where $S_2 > S_1$.

$$Q_{rev} = \int_1^2 Tds \;\; kJ \tag{3.11}$$

An isothermal process for which temperature is held constant is shown in

Figure 3-6 (b). Again, heat is into the system given by following.

$$Q_{rev} = T_2(S_2 - S_1) \; kJ \tag{3.12}$$



(a) Internally Reversable

(b) Reversable, Isothermal

Figure 3-6 T-S diagram for Internally reversible processes and cycles

3.7. Carnot Cycle

A special cycle often employed in Thermodynamics is called a Carnot cycle shown in Figure 3-7. This cycle is not physically realizable but serves as a benchmark for the theoretical limit regarding a heat engines efficiency. A Carnot cycle is a cycle that consists of two isothermal processes and two adiabatic processes connected together and returning to same state that it started. From 1 to 2 the process in an isothermal process with heat *into* the system, $S_2 > S_1$. From 3 to 4 the process in an isothermal process with heat *out* of the system $S_4 < S_3$. From 4 to 1 and from 2 to 3, since *ds* = 0 there is no heat transferred, Q = 0.

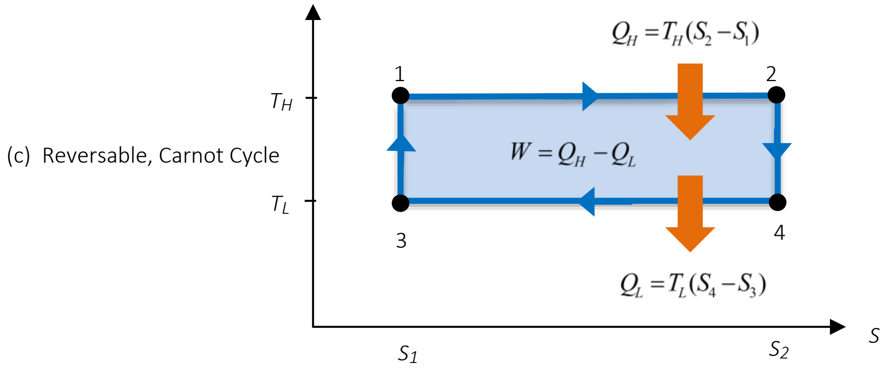


Figure 3-7 Carnot Cycle

For a Carnot Cycle the net-work done by or to the system is the shaded area of the rectangle 1-2-3-4. If the sign of *W* is *negative* the work is *into* the system. If the sign of *W* is *positive* the work is *out of* the system.

$$W = (Q_H - Q_L) \quad kJ \tag{3.13}$$

3.8. Enthalpy

Enthalpy is a property of a substance, like pressure, temperature, and volume, but it cannot be measured directly. Enthalpy measures the capacity of a fluid to do work. A fluid's ability to do work depends on both internal energy *U* and pressure *P*. When fluid with specific internal energy (*u*) leaves a control volume the system has to do flow work on it (*P v* per unit mass) and this energy is transported along with the fluid so that its energy per unit mass becomes *u* + *P v*. This combination of energies occurs so frequently in the study of thermodynamic systems it is given a special name, *enthalpy*, denoted by *H* [3]

$$H = U + P \cdot V \tag{3.14}$$

Where *U* is the internal energy and *P V* is the work associated with pressure and volume work. The *specific enthalpy* (enthalpy per unit mass) is

$$h = \frac{H}{m} = \frac{U}{m} + P \cdot \frac{V}{m} = u + P \cdot v \tag{3.15}$$

The change in enthalpy is given by

$$dh = du + vdP + Pdv \tag{3.16}$$

Enthalpy is very useful in various applications, particularly when studying the heating and expansion of gases. When we heat a constant-volume system, *P dv* = 0 so its internal energy increases *du*, which we observe as a rise in temperature *dT* and an increase in pressure *dP*. If the system is free to expand part of the energy supplied is lost in doing work to push back the surroundings. It therefore takes more energy to heat a system at constant pressure than at constant volume.

We also know from Gibb's equation [3]

$$Ts = du + dPv \tag{3.17}$$

Therefore, we can express enthalpy in terms of entropy

$$dh = Ts + vdP \tag{3.18}$$

The *enthalpy-entropy diagram* or *H-S diagram* is another useful tool used in the analysis of analysis of steady-flow devices such as nozzles, turbines and compressors. The vertical distance *Δh* represents the change in enthalpy (work) and *ΔS* represents change in entropy due to reversibilities.

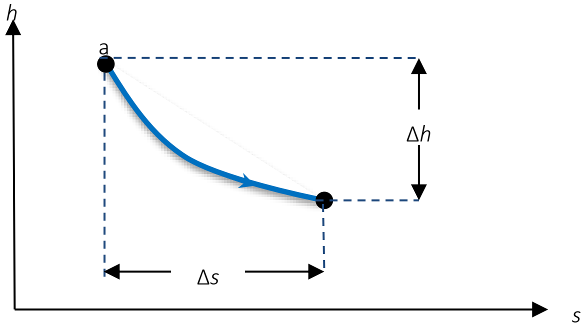


Figure 3-8 *H-S* diagram for steady flow systems

3.9. Heat

Heat is a commonly misunderstood concept, even among HVAC specialist. Heat is not a static entity or even a property of state, like mass, temperature or pressure. Heat Q, like its mechanical counterpart work W, is a transient interaction that ceases to exist once the process has ended. One can say an object has a given amount of mass, temperature and pressure. However, one cannot say whether an object contains or stores an amount of heat. An object contains internal energy U which is a measure of the molecular potential and kinetic energy. Temperature, however, is a measure of the molecular kinetic energy. When energy is in motion from point A to B, it becomes heat Q (thermal energy in transit). Recall the analogy described earlier. Heat is like rain and internal energy is like water. When water vapor is in a cloud its water. When the vapor condenses and falls it becomes rain (water in transit). Once the droplets hit the lake surface, they are no longer rain, but just water.

3.10. Sensible and Latent Heat

When an object is heated, its temperature rises, due to an increase in the average molecular kinetic energy. This heat is *sensible heat* since we can be sensed by a temperature change. For air this heat is referred to as dry heat and is measured by dry-bulb temperature i.e., standard thermometer.

When a substance changes state from solid to liquid or liquid to vapor, the absorbed energy is used to change the molecular bonding, spreading the molecules into a larger volume; but it does not change the average molecular velocity (kinetic energy), therefore temperature does not change. The heat added to or removed from a substance during a phase change is called *latent heat*, since the changes are hidden or concealed with respect to dry-bulb temperature. For air, latent heat represents the wet heat in the air that would be required for water to undergo a phase change to a vapor and is measured by wet-bulb temperature.

The terms *total capacity* (sensible and latent heat) and *sensible capacity* are used to define a unit's cooling capacity. Sensible capacity is the capacity required to lower the temperature and latent capacity is the capacity to remove moisture from the air. The percentage of the capacity that goes toward sensible cooling is called *Sensible Heat Ratio (SHR).* For example, a system that has an SHR of 70% and 10,000 Total BTUs of capacity would produce 7,000 BTUs of sensible cooling and 3,000 BTUs of latent removal.

The following figure shows the phases of water which contains a total of 1,352 BTUs of energy between 0° and 300 °. Of this total, it takes a whopping 970 BTUs of latent heat to vaporize the water. This is why refrigeration systems utilize latent heat of vaporization to expel/extract heat to/from the surroundings.

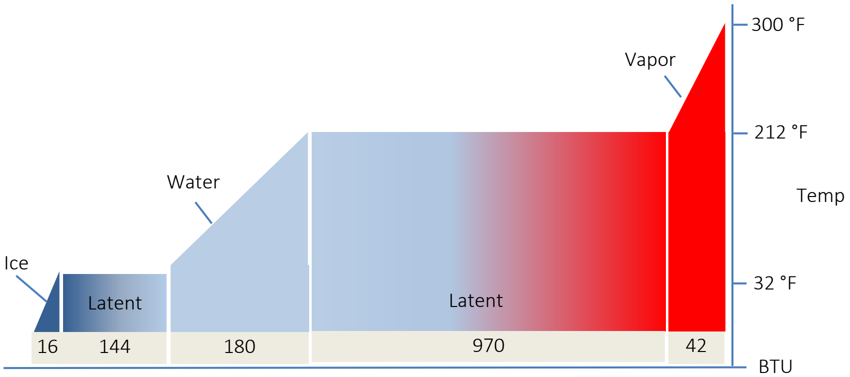


Figure 3-9
Sensible and latent heat for a pound of water (1,352 BTU/lb of water)

4. VAPOR COMPRESSION REFRIGERATION SYSTEM, VCRS

4.1. Carnot VCRS (Heat Pump)

The most frequently used refrigeration/Heat Pump cycle is the *Vapor Compression Refrigeration System (VCRS)*, which involves four processes: compression, condensing, expansion, and evaporation as shown in Figure 4-1. A review of this cycle is fundamental in understanding the Gas Absorption Heat Pump (GAHP) to be discussed later on.

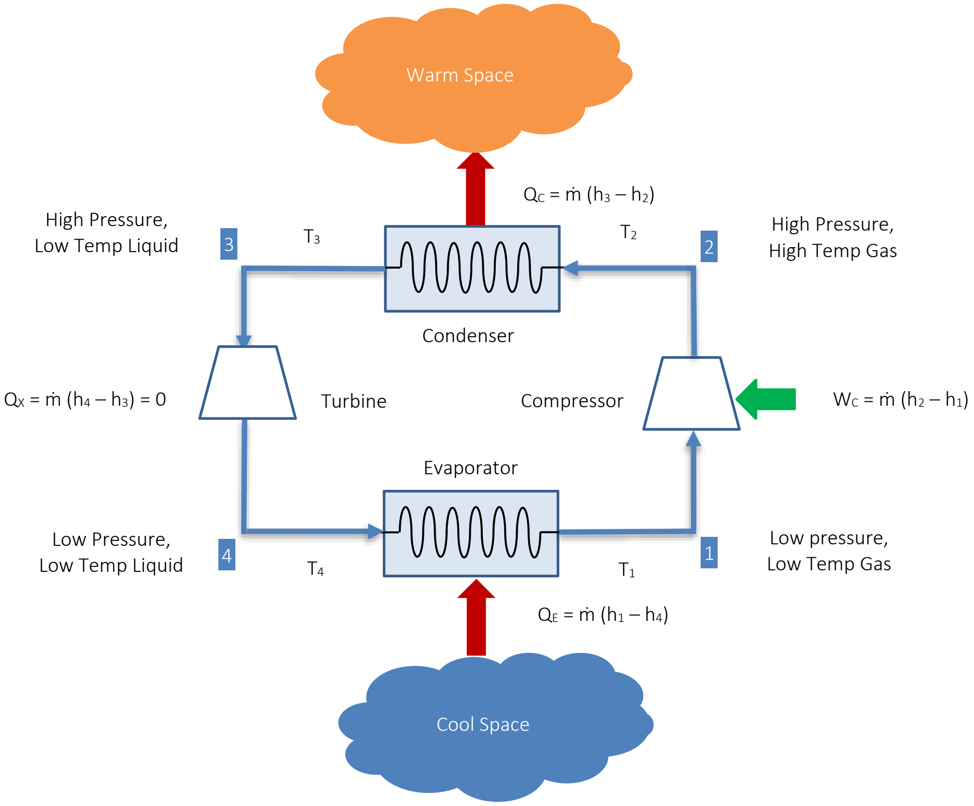


Figure 4-1 Idealized Vapor Compression Cycle (VCRS) used for refrigeration and heat pumps

4.2. Carnot Cycle for VCRS

It is useful to reference physically realizable systems to an ideal benchmark which is the Carnot Cycle (CC). We can visualize the Carnot Cycle (CC) using a standard *T-S* diagram shown in Figure 4-2.

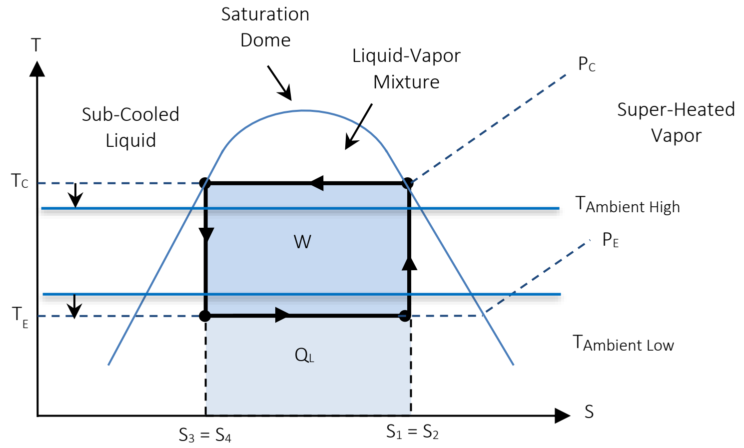


Figure 4-2 T-S diagram Ideal Carnot Vapor Compression Cycle

From points 4-1:

> The evaporator is an isothermal and isobaric process that extracts *latent heat* from the higher temperature surroundings and converts it to a vapor.

From points 1-2:

> The low temperature, low pressure gas is compressed isentropically, raising the temperature to the condenser temperature T_C and raising the pressure to the condenser pressure P_C. Since the compressor is not 100% efficient, heat will be lost during this process. It is assumed the compressor is adiabatic, so there is no heat lost to the surrounding ambient. Therefore, the lost heat is transferred to the refrigerant, resulting in an increase in *enthalpy* at state 2. The work required by the compressor is proportional to $h_2 - h_1$. *This work is the work needed to drive the entire cycle*!

From points 2-3:

> The high pressure, high temperature saturated vapor is condensed, converting the vapor to a liquid and expelling the *latent heat* in the refrigerant isothermally to the cooler surroundings.

From points 4-3:

> The high-pressure saturated liquid then flows through a turbine and undergoes isentropic expansion reducing the temperature and pressure to T_E and P_E.

We can visualize a refrigeration or heat pump cycle as temperature lifting system. In the low region, the temperature of the refrigerant is higher than ambient and absorbs latent heat. In the high region, the temperature of the refrigerant is higher than ambient and expels latent heat.

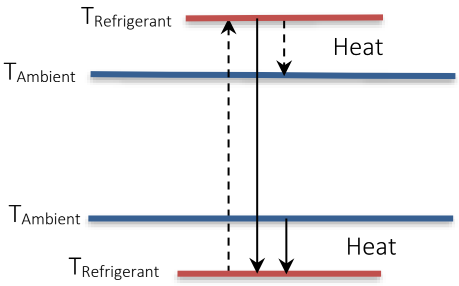


Figure 4-3 T-S diagram for an Vapor Compression Refrigeration System (VCRS) or Heat Pump

All processes in the Carnot cycle are reversible. In reality, this is not physically realizable, so the Carnot cycle serves as a theoretically ideal benchmark.

Reversibility:

> In an isolated system, the second law states: without external energy, a process that reduces the entropy of the system is not possible. Assume a closed system in which a process in the forward direction from A to B increases entropy. If the process were to follow the reverse path B to A; in order to return to its initial thermodynamic state, it would require a decrease in entropy which is impossible by virtue of the second law. We say this process is *irreversible*.
>
> If in a closed system a process from A to B does not increase the system entropy, then it is possible for the system to follow the reverse path from B to A and return to its initial thermodynamic state. In this case we call the process *reversible*.
>
> To illustrate, consider rubbing your foot on a carpet from point A to B. You feel the heat due to friction during this forward path. If you were to then rub your feet on the carpet from point B to A, in the reverse direction you would not be able to restore the lost thermal energy, as such, this process is irreversible due to friction. Reversibility inevitably reduces the efficiency of any closed system.

The Carnot cycle can be analyzed using the *T-S* diagram in Figure 4-4. From equation 3.11, the heat transfer Q_H for the process from 2 to 3 is the area 23AB. The heat transfer Q_L for the process from 4 to 1 is the area 41BA. Therefore, the total work for the cycle is given by the area 1234.

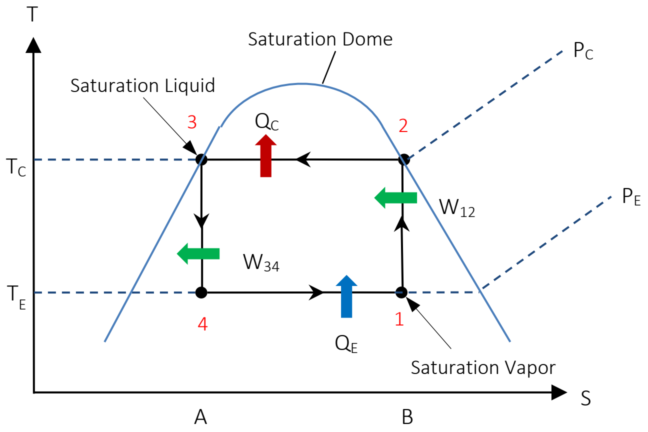


Figure 4-4 T-S diagram Ideal Carnot Cycle

4.3. Carnot VCRS Cycle with Dry Compression

There are several issues with the ideal Carnot Cycle shown in Figure 4-1.

1. It is impossible to build a machine based on a Carnot Cycle due to inherent reversibilities
2. As seen in Figure 4-4, the compression process from 1-2 contains a mixture of liquid and vapor. This is referred to as wet compression. However, the presence of any liquid in the compressor can ruin the compressor.
3. It is not economically justified to use a turbine (from 3 to 4) for isentropic expansion of liquid refrigerant

We can achieve dry compression in Carnot refrigeration cycle that eliminates the wet compression issue as shown in Figure 4-5 [6]. This system with dry compression consists of one isentropic compression process (1-2) from evaporator pressure P_E to an intermediate pressure P_I and temperature T_C followed by an isothermal compression process (2-3) from the intermediate pressure P_i to the condenser pressure P_C. This modification removes wet compression; however, the condensation process being isothermal requires two compressors. One for P_E to P_I and another for P_I to P_C. This is a highly undesirable solution! In the next section, we introduce a practical VCRS that addresses these issues in a pragmatic manor.

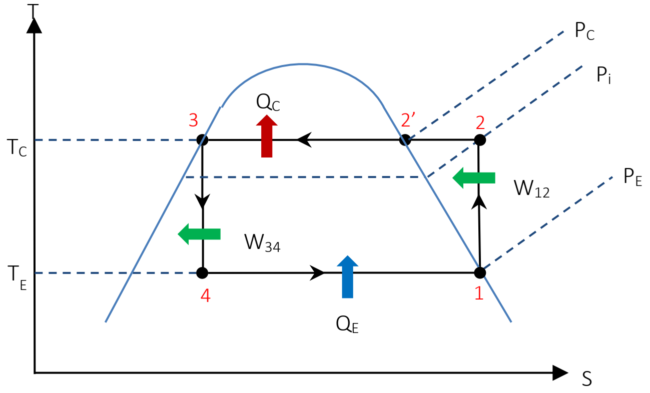


Figure 4-5 T-S diagram Carnot Vapor Compression Cycle with Dry Compression

5. PRACTICAL VCRS (HEAT PUMP)

5.1. Overview

The difficulties described in the previous section can be eliminated if: (1) the isothermal heat rejection from 2 to 3 in Figure 4-5 is replaced with isobaric heat rejection, and (2) If the isentropic expansion (turbine) from 3 to 4 is replaced with an isenthalpic throttling device such as a Joule-Thompson valve (JTV). A practical VCRS with these modifications is shown in Figure 5-1.

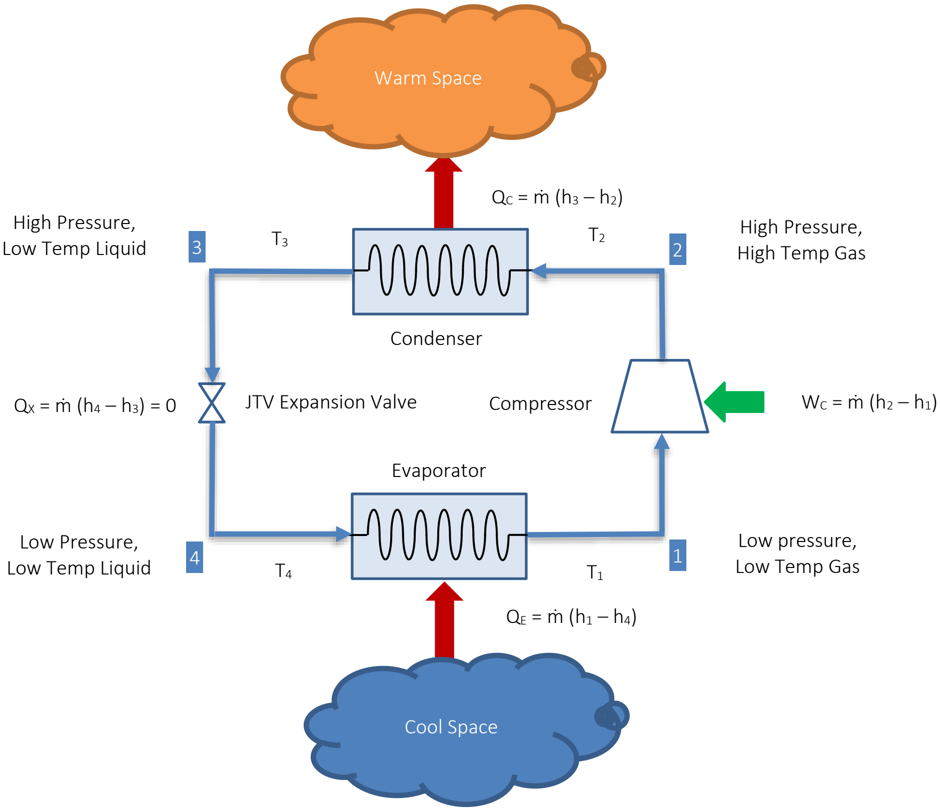


Figure 5-1 Practical Vapor Compression Refrigeration System (VCRS)

5.2. VCRS *T-S* Diagram

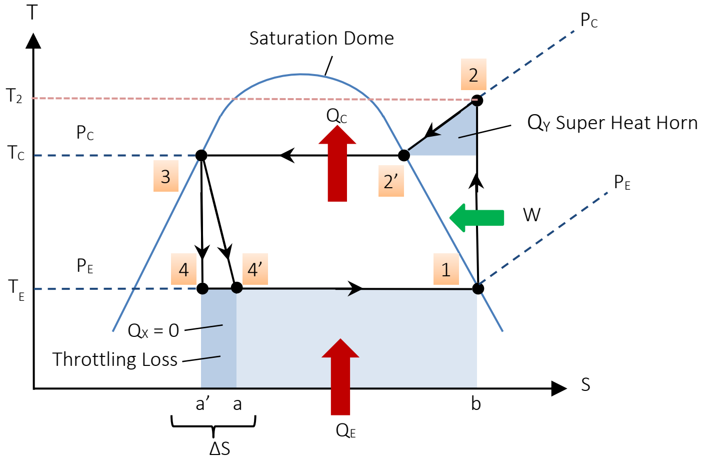
The T-S diagram for a single stage Vapor Compression Cycle (VCRS) is shown in Figure 5-2. From this diagram, we can identify the following 4 processes.

1-2: Isentropic and adiabatic dry compression of saturated vapor

2-3: Isobaric heat rejection

3-4: Isenthalpic and adiabatic expansion of saturated liquid

4-1: Isobaric and isothermal heat absorption



NOTE:
safety margin of 8° Super Heat is
typically maintained in most designs

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Figure 5-2 T-S diagram for a practicle Vapor Compression Heat Pump Cycle (VCHP) [6]

In this ideal Vapor Compression Heat Pump Cycle (VCHP), the saturated vapor at state 1 is compressed isentropically to the condenser pressure P_C at state 2, eliminating the extra compressor. The temperature T_2 at state 2 is above the external warm ambient. The superheated refrigerant vapor is then condensed isobarically to a saturated liquid at state 3. Next, the saturated liquid is expanded isenthalpically, replacing the turbine with a simple expansion valve. The temperature of the refrigerant at state 4 drops below the cooler ambient. Recall that both the condensation and evaporator processes are reversible for either Carnot or practical VCHP cycles.

Evaporator:

The evaporator energies for a Carnot cycle and a VCHP cycle is given by

$$q_{E,Carnot} = \int_{4'}^{1} TdS = area\{4'1ba'\} \qquad \text{Evaporator Heat (Carnot)} \qquad 5.1$$

$$q_{E,VCRS} = \int_{4}^{1} TdS = area\{41ba\} \qquad \text{Evaporator Heat (VCRS)} \qquad 5.2$$

Condenser:

The evaporator energies for a Carnot Heat Pump (CHP) and a Vapor Compression Heat Pump (VCHP) are given by

$$q_{C,Carnot} = -\int_{2''}^{3} TdS = area\{\,2''3a'b\,\}$$

Condenser Heat (Carnot) 5.3

$$q_{C,VCRS} = -\int_{2''}^{3} TdS = area\{\,234ab\}$$

Condenser Heat (VCRS) 5.4

Expansion:

The expansion device is isenthalpic but not isentropic, thus there is a decrease in the heat that can be absorbed by the evaporator. This is known as the *throttling loss*. The entropy increases in a throttling device due to the irreversibility introduced by turbulence. This is analogous to friction. Turbulence is not present in a turbine as its flows are laminal. As such, a turbine is often modeled as an isentropic process.

$$q_X = \int_{4}^{4'} TdS = area\{44'aa'\}$$

Throttling Loss (VCRS) 5.5

Super Heat:

The increase in heat rejection rate of VCHP compared to the Carnot cycle is equal to the area {2’’ 2’ 2}. This region is known as *superheat horn* [6]. The increase is due to the replacement of an isothermal heat rejection in the Carnot system Figure 4-5 with and isobaric heat rejection in the VCHP Figure 5-1.

$$q_Y = -\int_{2''}^{3} TdS = area\{2''22'\}$$

Super Heat (VCRS) 5.6

5.3. VCRS versus VCRS (Carnot)

The work of the Carnot and VCHP cycles is

$$w_{Carnot} = (q_C - q_C)_{Carnot} = \ area\{\ 12''3\ 4'\}$$

Work (Carnot) 5.7

$$\begin{aligned} w_{VCHP} &= (q_C - q_C)_{Carnot} \\ &= \ area\{22'2''\} + area\{2''34'1\} + area\{44'a'a\ \} \end{aligned}$$

Work (VCRS) 5.8

The increase in work input in VCHP compared with an ideal Carnot heat pump is

$$\begin{aligned} \Delta w_{VCHP} &= w_{VCHP} - w_{Carnot} \\ &= \ area\{22'2''\} + area\{44'a'a\ \} \end{aligned}$$

Work (VCRS) - Work (Carnot) 5.9

5.4. Throttling Loss

We stated that the expander was isenthalpic, which we now prove.

Isenthalpic expansion, also known as Joule-Thomson expansion, is a process where a gas or liquid at pressure P_1 flows from region-1 to a region-2 of lower pressure P_2.

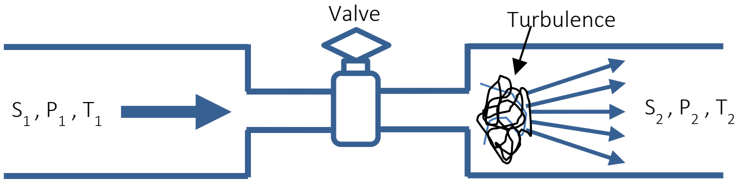

Figure 5-3 Joule-Kelvin expansion through a throttle [11]

During the expansion of the gas there is a turbulence occurring which increases the entropy. Hence the system is not isentropic.

$$S_2 > S_1 \tag{5.10}$$

Throttling valves are often modeled as nearly adiabatic even if not insulated because their external surface is small. No work is done by the gas in a throttling process and no heat is added or lost (ideal case), therefore the energy balance equations are

$$dh = dq - dW = 0 + 0 = h_2 - h_1 \tag{5.11}$$

From which

$$h_2 = h_1 \qquad \text{Q.E.D.} \tag{5.12}$$

5.5. Worked Example

The P-H diagram for R134a refrigerant is shown in Figure 5-4. Isotherms are shown as blue lines and isentropes (lines of constant entropy) are shown in light blue.

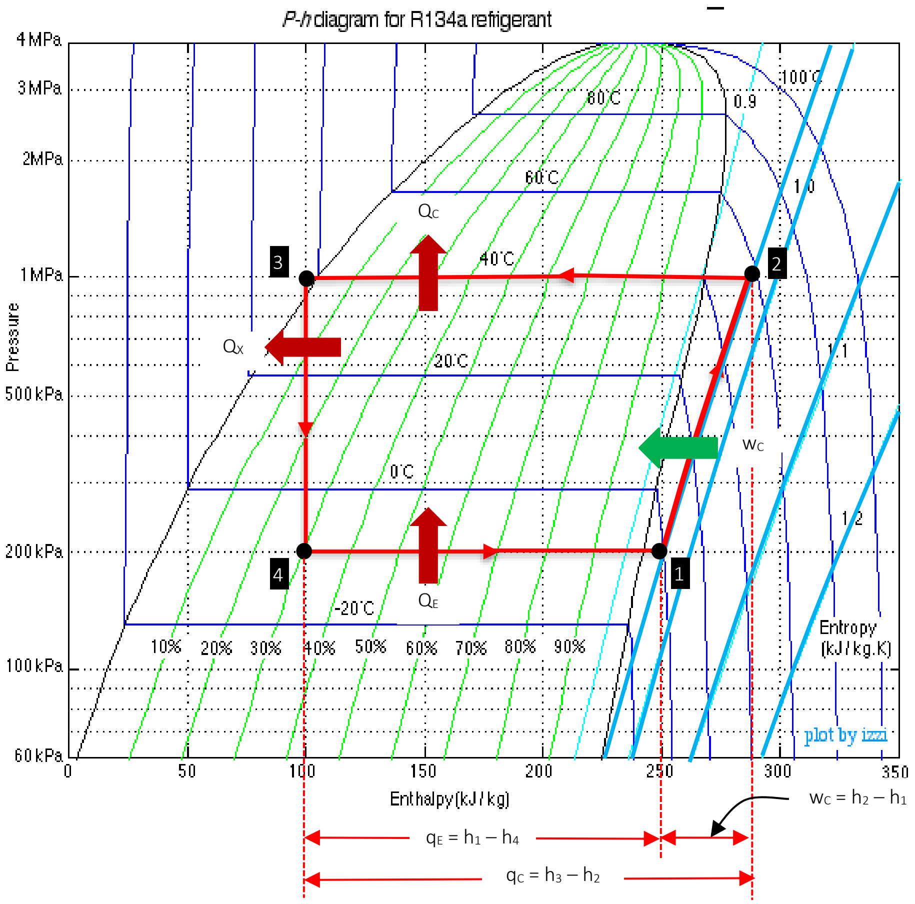

Figure 5-4 P-H diagram for R134a refrigerent

The component energies for the VCHP are:

$$w_C = h_2 - h_1 = (290 - 250) = 40 \quad kJ/kg$$ Compressor Work 5.13

$$q_C = h_3 - h_2 = (100 - 290) = -190 \quad kJ/kg$$ Condenser Heat 5.14

$$q_X = h_4 - h_3 = (100 - 100) = 0 \quad kJ/kg$$ Expansion Heat 5.15

$$q_E = h_1 - h_4 = (250 - 100) = 150 \quad kJ/kg$$ Evaporator Heat 5.16

The heat pump efficiency is

$$\eta_{HP} = \frac{q_C}{w_C} = \frac{190}{40} = 450\%$$ 5.17

Ratio of heat in and heat out is approximately 80%

$$\frac{q_E}{q_C} = \frac{150}{190} \approx 80\%$$ 5.18

6. GAS ABSORPTION HEAT PUMP (GAHP)

The key differences between a heat pump based on a Vapor Compression Refrigeration System (VCRS) and Gas Absorption Heat Pump (GAHP) are:

1. A GAHP uses a thermally driven compressor not a mechanical compressor
2. A GAHP makes use of low-grade renewable energy (Ambient Air) to supplement its non-renewable energy

A description of the GAHP system is given by [10]. "Ammonia vapor leaves the evaporator and enters the absorber, where it dissolves and reacts with water to form NH_3+H_2O. This is an exothermic reaction; thus, heat is released during this process. The amount of NH_3 that can be dissolved in H_2O is inversely proportional to the temperature. Therefore, it is desirable to cool the absorber to maintain its temperature as low as possible. The liquid NH_3+H_2O solution is rich in NH_3 and is then pumped to the generator. Heat is transferred to the solution from a source to vaporize some of the solution. The vapor is rich in NH_3 and passes through a rectifier used to separate the water and return it to absorber. The high-pressure pure NH_3 vapor then continues to the condenser. The Hot NH_3+H_2O solution, which is weak in NH_3, then passes through a regenerator (absorber), where it transfers some heat to the rich solution leaving the pump, and is throttled to the absorber pressure.

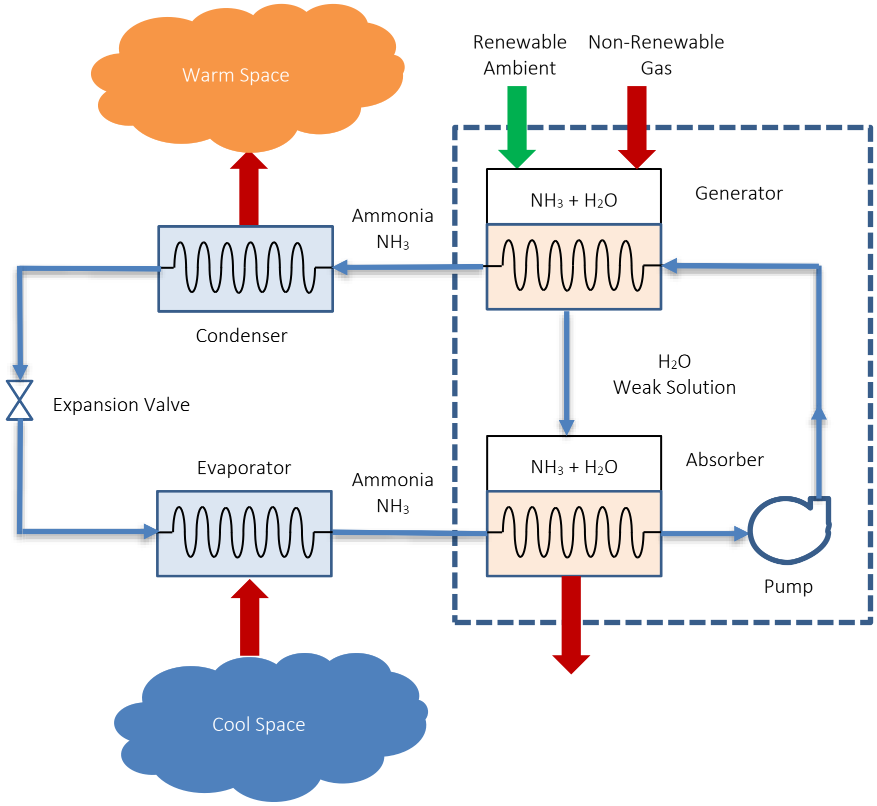

Figure 6-1 Simplified diagram of a Gas Absorption Heat Pump

In vapor compression refrigeration systems, the vapor is compressed mechanically using the compressor, whereas in an absorption system the vapor is first converted into a liquid and then the liquid is pumped to condenser pressure using the solution pump. For the same pressure difference, the work input required to pump a liquid is much less than the work required to compress a vapor due to the small specific volume of liquid.

$$w_P = -\int_{P_E}^{P_C} v\,dp \qquad \text{pump work} \qquad 6.1$$

Unlike the VCRS the GAHP pump-work is often negligible compared to the generator heat input. Therefore, the efficiency for heat pumps based on VCRS and GAHP are approximately:

$$COP_{VCRS} = -\frac{Q_E}{W_C} \qquad 6.2$$

$$COP_{GAHP} = -\frac{Q_E}{Q_G} \qquad 6.3$$

The GAHP system involves only heat terms, so it is often referred to as a *heat driven system*.

A simplified diagram for a GAHP system shows typical energy flows. In this case we see an efficiency of 150%. GAHP system efficiencies range from 120% to 180% or higher depending on design.

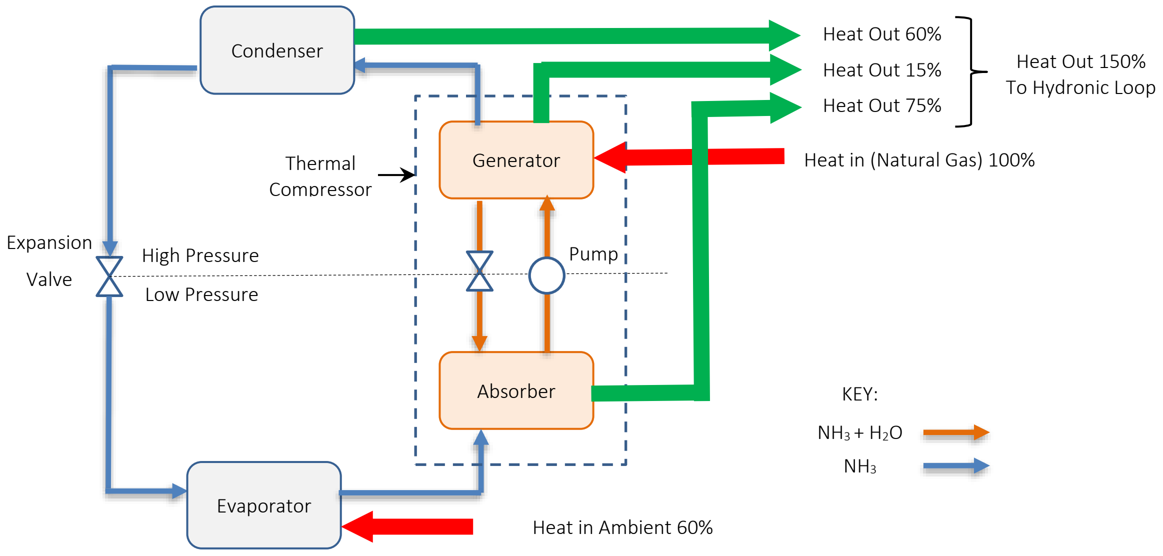

Figure 6-2 Simplified Gas Absorption Heat Pump [12]

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